Moon Project Handout

Summary: You will recreate and interpret the geometric and timing measurements performed by the Ancient Greeks in order to determine the sizes of the Sun, Moon, and Earth and the distances between them. You will also measure the angular motion of the Moon and, jumping ahead in history to make use of Newton's laws, determine the Earth's mass from the speed and radius of the Moon's orbit.

I: A Mental Model of the Sun, Moon, and Earth (Do in class.)

Your teacher will provide a demonstration to illustrate how the motion of the Moon around the Earth leads to both Moon phases (full moon, half moon, etc.) and lunar eclipses. Watch out! Many students think Moon phases and eclipses are both caused by the Earth shadow, but in reality only one is. (Which?)

After the demo, test your understanding: Which of these images could represent an eclipse phase but NOT a monthly Moon phase? (Discuss with your team; then check your answer with your teacher.)

A B C D

II. Measuring the Radius of the Earth (Do with your team.)

How big is the Earth? Nowadays we can look this up in a book, but how was the measurement made in ancient times? You can actually estimate the radius of the Earth with no equipment but a stopwatch and some math. Here's how:

The Earth takes a day to rotate once. Its rotation doesn't speed up or slow down, so that means that every second the Earth rotates by a certain fixed angle. Calculate this angle as follows:

- How many degrees are in one rotation? (Hint: think of a circle)
- How many seconds are in a day?

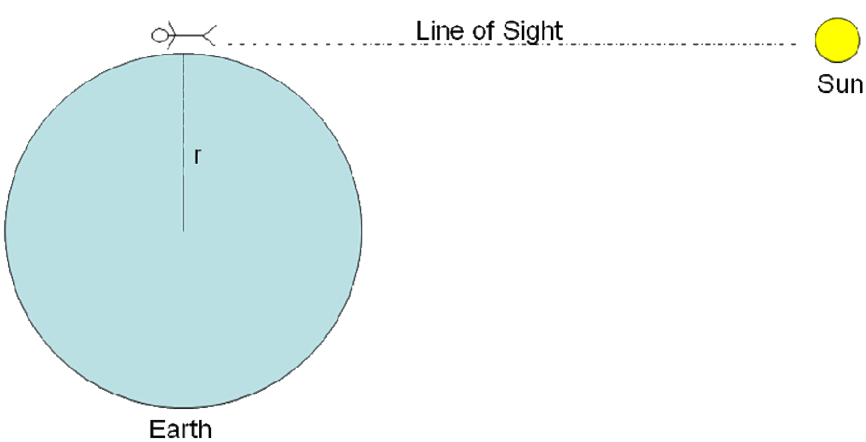
- Now set up a ratio: $\frac{\text{1 second}}{\text{seconds in a day}} = \frac{\text{degrees in a second}}{\text{degrees in a day}}$

  You know everything except one thing in this equation. Solve for it by cross multiplying. Compare with teammates, correct your work if necessary, then put your final answer in the box.

Degrees the Earth turns per second:

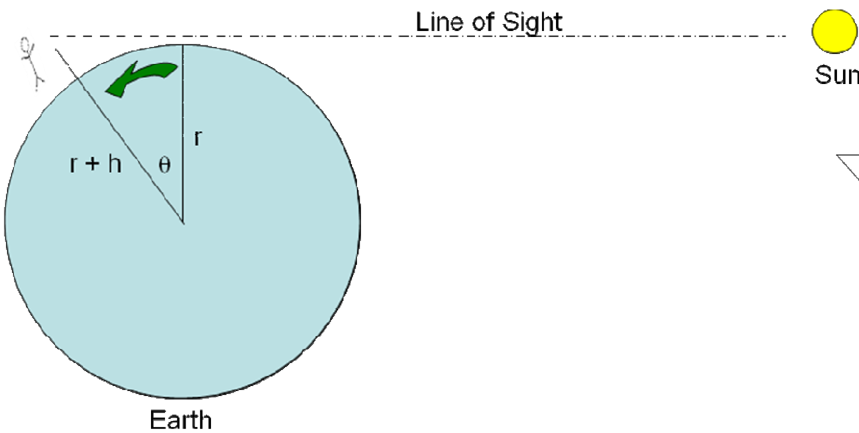
You now know how many degrees the Earth turns every second, its "angular speed." (See Appendix.) But how does this help in measuring the radius of the Earth? Say you're lying down on the lawn watching the Sun set on a peaceful summer evening. Just as the Sun sets, you'd see something like this picture:

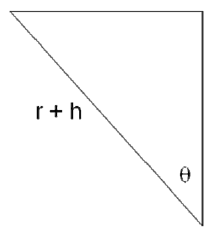
The diagram below shows the same situation, only from the perspective of someone in space. You're lying down on the surface of the earth, and your line of sight makes a straight line with the Sun. We see the Sun set because the Earth rotates. In the diagram it rotates counterclockwise, so after a short amount of time it will be nighttime and you will not be able to see the Sun at all.



What would happen if you stood up just after the Sun set? You'd see something like the picture below. Because you're now looking at the horizon from a slightly higher point, you'd be able to see a small sliver of Sun.

The diagram below shows this situation. As we watch the sun set for the second time, the Earth rotates by a certain angle, θ.





If we can find this angle, we can use trigonometry to **solve for the radius of the Earth, which was our original goal.** (See diagram above.) We find the angle by timing how long it takes the Sun to set again once we stand up. Then, we can use the angular speed of the Earth's rotation to convert the time into an angle.

In order to make this measurement we need an unobstructed horizon (which is not so easy to find). Instead of trying this yourself, assume that it takes about 11 seconds for the Sun to set after you stand up.

- How many degrees does the Earth rotate in this amount of time? This will be angle θ in diagram 3. (Hint: Use your earlier estimate of the angular speed of the Earth's rotation.)

- What is your height (in meters)? This will be the value *h* in diagram 3.

- Now, use trigonometry to relate the angle to the lengths in the triangle. Use symbols, and don't plug in numbers just yet.

- Can you find a way to rearrange your equation to solve for the radius of the Earth *r*? When you have an expression for *r*, plug in your measured values to calculate *r*.

- Look up the actual value of the radius of the Earth and compare it to your estimation. If necessary, correct any errors in your calculation.

- What about this measurement most likely produced the greatest error in the result? Can you think of a more precise way to measure the radius of the Earth?

- Put your best, final answer in the box below.

Radius of the Earth:

- As a review, summarize what key measurements are needed to find the radius of the Earth, and explain how they relate to the triangle diagram on page 3.

III. Measuring the Angular Sizes of the Moon and Sun (Do with your team.)

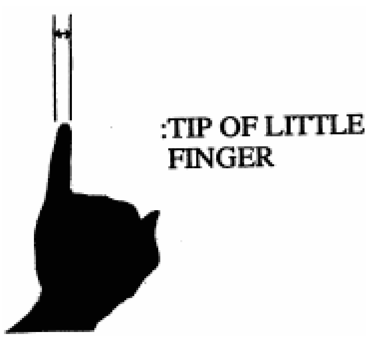
Held at arm's length, the tip of your little finger is roughly 1° in width. Using this approximate measuring tool, estimate the angular sizes of the Sun and the Moon. **DO NOT STARE AT THE SUN!** It is best to estimate the Sun's size when it is behind a cloud or low in the sky near sunset. Average your team's answers to get best estimates and put them in the boxes below.

My Answers:

Sun ______ degrees

Moon ______ degrees

Team Averages:

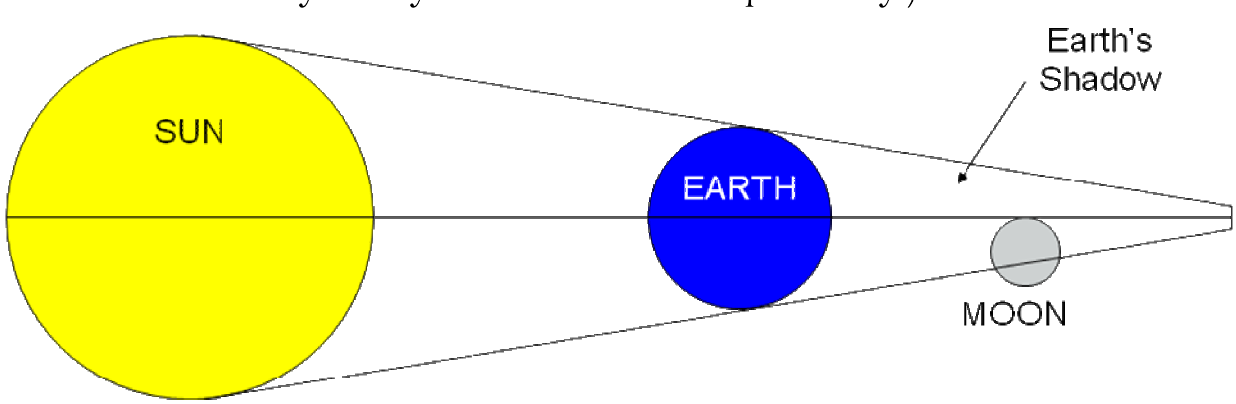


Angular size of the Sun:
Angular size of the Moon:

IV. Measuring the Actual Size & Distance of the Moon (Do with your team.)

Aristarchus's Lunar Eclipse Measurement

Lunar eclipses were among the most important astronomical events in the lives of the early Greeks. Aristarchus used his understanding of the arrangement of the Earth–Moon system during a lunar eclipse and some basic geometry to determine the relative sizes of the Earth and Moon. (We won't ask you to recreate this measurement since you may not have a lunar eclipse handy!)



A lunar eclipse occurs when the Moon passes through the shadow of the Earth. Because the Earth is larger than the Moon, it takes the Moon some time to pass completely through the shadow. By measuring this length of time, Aristarchus was able to determine that the diameter of the Earth was roughly three times that of the diameter of the Moon. More modern estimates give a ratio of 3.7. Using this ratio, calculate the size of the Moon from your answer to part II. Watch out for the factor of 2 between radii and diameters!

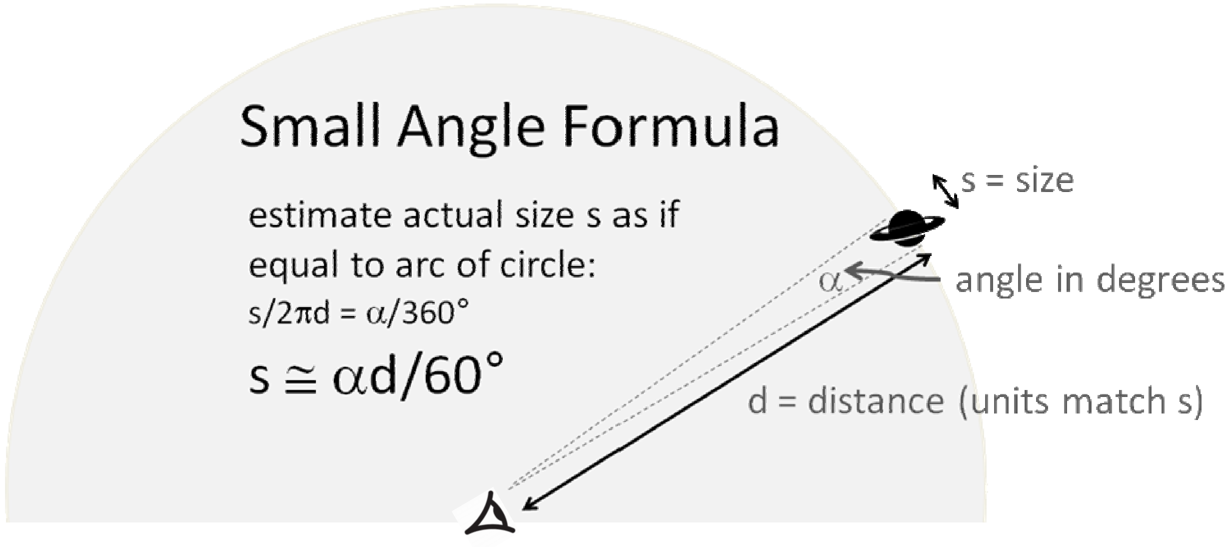
Compare with your teammates, correct your work if necessary, and put your final answer in the box below.

Actual size (diameter) of the Moon:

The Small Angle Formula

We can now use the angular and actual sizes of the Moon to find the distance to the Moon. The key is to apply the fact that more distant objects appear smaller.

The small angle formula relates the angular sizes of objects to their distances, making the approximation that the actual size s of an object (its diameter) is equal to the arc length it spans along a circle whose radius is the distance d from the observer to the object. This formula works pretty well even for large objects like the Moon. (See the diagram on page 7.) Use the small angle formula to determine the distance to the Moon.



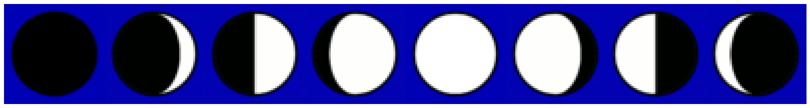
Compare with your teammates, correct your work if necessary, and put your final answer in the box below.

Distance to the Moon:

V. Measuring the Phases and Motion of the Moon (Do with your team.)

To measure the size and distance of the Sun, as well as the mass of the Earth, we must first track the Moon's orbit. Over a period of about a month, your team should observe the phase of the Moon nightly, in order to determine both the period of its orbit and when exactly the Moon is half full (this occurs twice in each orbit). Divide up the days between you so that each student is responsible for several nights' observations. Don't worry if it's cloudy sometimes – you will see the pattern emerge from the clear nights over time. Use the log sheet on page 8.

Lunar Observing Log



New Crescent Half Gibbous Full Gibbous Half Crescent

Directions: Record the date, time, weather (e.g. clear, light clouds, etc.), and name of the student observer, along with a shaded drawing of the Moon phase he/she observes. Shade the *dark* part of the Moon: a New Moon should be shaded over the entire circle to show that it is completely dark and a Full Moon should be left unshaded to show that it is completely lit.

Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name
Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name
Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name
Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name	Date/Time Weather Name

Your log should allow you to determine when the Moon is half full.

Date when the Moon is half full:

You will use this date in part VI.

Your log should also reveal the period of the Moon's orbit in days.

Period of the Moon's orbit:

From the period, you can compute the Moon's orbital speed. First, figure out how many degrees the Moon travels through in one orbit. (Hint: this is easy!) Now, compute its angular speed in degrees/day.

Angular speed of the Moon's orbit:

To compute the Moon's orbital speed in meters/second (referred to as its "linear" speed), see the Appendix for a refresher on how to convert from angular to linear motion. You'll want to convert the angular speed to radians/second, and you'll need to make use of the Earth-Moon distance found in part IV – which is the radius of the Moon's orbit.

Linear speed of the Moon's orbit:

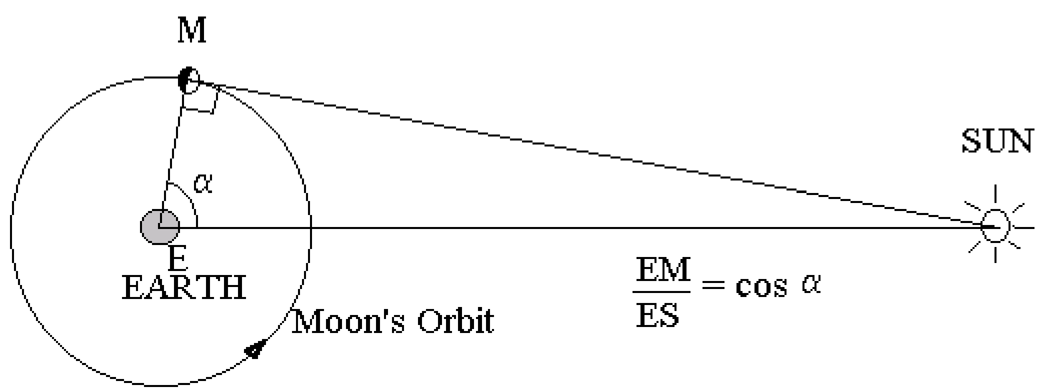
Think about how Newton's Laws would explain this motion. What keeps the Moon going in its orbit? How would the Moon move if the Earth were to suddenly disappear, removing the force of gravity on the Moon?

We know that the Moon does not fall down to the ground, and that its orbital speed is constant. Yet the Moon feels gravity from the Earth. Explain how the force of gravity affects the Moon's motion.

VI. Measuring the Actual Size & Distance of the Sun (Do with your team.)

Aristarchus's Sun-Moon Angle Measurement

Aristarchus determined the distance to the Sun by measuring the angle α in the diagram below, when the Moon was precisely half full. Write down a trigonometric equation relating the Earth-Sun distance and the Earth-Moon distance to α.



In principle, it is easy to measure the angle α between the Sun and Moon, because both are simultaneously up in the sky at half-moon phase. Mark the day side of the Earth in the diagram to see why this is true.

In practice, measuring α accurately is difficult. Therefore, obtain α from your teacher for the date you found in part V, and write it here.

Moon-Sun angle at half-moon phase:

Now using the Earth-Moon distance you found in part IV, solve for the distance between the Earth and the Sun.

Distance to the Sun:

Finally, use the small angle formula to compute the diameter of the Sun from its distance and angular size (part III).

Actual size (diameter) of the Sun:

VII. Historical Reflection (Do on your own.)

So far, we have been able to recreate some measurements that were first made over two millenia ago. We found the radius of the Earth, not with a giant meter stick, but with some basic trigonometry. The size and distance of the Moon came from a lunar eclipse measurement – not easy to recreate on demand, but requiring no special equipment. We obtained the Moon's orbital speed from naked-eye observations of its phases. Using some more trigonometry at the half-moon phase, we computed the size and distance of the Sun (no small task!). We now know the absolute sizes and distances of the three most important objects in our solar system. Please take a minute to be amazed by the fact that we did not need any astronomically sized measuring device to do all this. It just took a little bit of geometry.

Because of the vast size and distance of the Sun, as well as other evidence (such as the retrograde motion of the planets), Aristarchus became convinced that the Earth could not be at the center of the Universe. The rest of the ancient Greeks were not persuaded however – for one thing, it seemed nutty to suggest that the Earth is moving around the Sun, when no one can feel it move. Only many centuries later would Copernicus, Kepler, and Galileo vindicate Aristarchus.

1. Consider all of the different measurements that Aristarchus made in both the lunar eclipse measurement and the Sun-Moon angle measurement. Which of those measurements contain the largest possibility for error? Which result(s) would be most affected by that error?

2. What implications, if any, did Aristarchus's measurements have for the people of his time (think socially, politically, economically, religiously)?

3. Explain how common sense, cultural values, and uncertainty about the validity of Aristarchus's measurements could affect the reception of his ideas. How is this situation similar and different from controversial scientific measurements being made today (e.g., CO_2 levels in the atmosphere and global warming, etc.)?

VIII. The Mass of the Earth (Do on your own.)

Aristarchus never measured the mass of the Earth, but he had all the ingredients he needed. It took Newton to figure out the laws of physics that would allow people to determine the Earth's mass from the Moon's orbital radius and velocity.

Finding the mass of the Earth is not like finding the mass of an apple. We do not have a special balance on which the Earth can fit. Therefore, we must use the physicist's definition of mass (inertia) to determine the Earth's mass from its effect on other objects.

Invoking the Laws of Physics

Using the following three equations and the measurements made in previous sections, determine the mass of the Earth.

$F_g = \frac{Gm_1m_2}{r^2}$	$F_{12} = -F_{21}$	$F_c = \frac{mv^2}{r}$

Start with a picture of the action/reaction force pair of the Earth – Moon system. Be sure to label information such as the velocity of the Moon that we calculated in the previous parts.

Appendix: Angular Motion

To describe the rotation of objects, we sometimes talk about angles instead of linear distances. Just as we can say how far an object has moved along a straight line, we can refer to *angular motion* to describe how much an object has rotated. We typically use the symbol θ (theta) to represent this angle.

Concept Check:
In which clock has the big hand rotated through the largest distance (a.k.a. "linear" distance) since twelve midnight? In which clock has the big hand rotated through the largest *angular* distance since twelve midnight?
A) a wristwatch showing 12:10 AM
B) a wristwatch showing 12:15 AM
C) a wall clock showing 12:10 AM

The "angular velocity" or "angular speed" ω is how fast an object is rotating – that is, how many radians (or degrees) it turns every second. So just as $v = d/t$ for linear motion, $\omega = \theta/t$ for angular motion, with units of rad/sec (or ^{o}/sec)

Concept Check:
In which pair does the second item have greater speed (a.k.a. "linear" speed)?
In which pair does the second item have greater *angular* speed?
A) the big hand on a wristwatch and the big hand on a wall clock
B) the big wheel on a tricycle and the little wheels on a tricycle

If you want to convert angular distances and speeds into linear distances and speeds, the key is to remember that the arc length along a circle of radius r is just $r\theta$ if θ is measured in radians. So the linear speed is $v = r\theta/t$, assuming we have converted angles from degrees to radians using 2π rad = 360^o.

Concept Check:
What is the angular speed of the big hand of a clock in degrees per minute? In radians per minute (you may approximate $2\pi=6$)? If a wristwatch has a big hand 1 cm long, what is the linear speed of the big hand?