Module 4: Motions Find Mass

Objectives/Key Points

Students will be able to:
1. Identify situations in which the centripetal acceleration condition applies, based on understanding the assumptions that go into its derivation.
2. Derive the Motions Find Mass equation from Newton's laws and the centripetal acceleration condition.
3. Apply the Motions Find Mass equation to measure enclosed mass using orbital parameters.

Unit Home

Circular Motion and Newton's Laws

Prerequisites

Students should be familiar with circular motion and Newton's laws. They should know that the distance in the Universal Law of Gravitation is measured between the centers of the masses. See Appendix if students have not previously seen the derivation of the centripetal force equation.

Time: 60-75 min depending on whether Appendix is used (+ optional 20 min post-lesson)

Materials

Graph paper with pre-marked axes
Weight on string

Sticking Points

1. Students tend to think that an equation is generally applicable, i.e. "always true." This lesson emphasizes that the Motions Find Mass equation is applicable under specific conditions, which the teacher should reinforce.

2. Students tend to generalize variables like V, R, M to "velocity," "radius," and "mass" without thinking too much about <u>*which*</u> velocity, radius, or mass. The teacher should emphasize this repeatedly.

3. Going from #5 to #6 below, students forget that they know the mass of the Sun.

Warm-Up

Ask students "Why is it difficult to find the masses of stars, planets, and asteroids?" Discuss the fact that there is no cosmic scale on which to weigh astronomical objects. Explain that students will now learn how astronomers measure masses, using the "Motions Find Mass" equation.

Pre-Lesson

1. Demo: Teacher swings mass around on a string (carefully!). Teacher should illustrate that an increasing force is required on the string as object swings faster. If desired, have students try the demo. Then complete concept checks (a) and (b).

In each situation, analyze the conditions required for a stable circular orbit. [Have students write down an answer for each, then discuss with partner, then discuss as a class.]

(a) If the central object is replaced with a more massive object, how must the orbital velocity differ if the orbital radius is to be the same?

Answer: The velocity must increase to keep the same orbital radius with a more massive central object.

(b) If the velocity of the orbiting object is reduced, how must the mass of the central object differ if the orbital radius is to be the same?

Answer: The mass must be decreased to keep the same orbital radius at lower velocity.

Main Lesson

2. Teacher will review that centripetal force mV^2/R is the force necessary to maintain a stable circular orbit balancing gravity. (Note If either the concept of a "derivation" or the idea that $F=mV^2/R$ is a derived equation is not familiar, teacher should insert optional short lesson on this here – see Appendix below). Then complete the following concept checks.

Assess whether we can apply the equations listed in each situation (have students first write yes or no for each entry, then discuss with a partner, then discuss as a class):

	$F=GMm/R^2$	$F=mV^2/R$
Teacher draw diagram of circular orbit		
… diagram of cometary orbit		
… diagram of falling object		
… diagram of something like the demo		

Answers: first three are yes for $F=GMm/R^2$, first and last are yes for $F=mV^2/R$; all rest no (note since gravity is involved in creating the string tension in the demo, they could say that $F=GMm/R^2$ is involved there too, just indirectly)

3. For a situation where both the Universal Law of Gravitation and the centripetal force condition apply, have the students attempt to derive an equation for the central mass M in terms of the V and R of the orbiting body ($M=V^2R/G$). After some time struggling, they can get help from the teacher. Label the result the "Motions Find Mass" equation.

4. Stress that the Motions Find Mass equation is a useful tool in astronomy. Give some verbal examples, for example what do you need to know about the Earth in order to measure the mass of the Sun. Discuss why you don't need to know the mass of the Earth.

5. Teacher should walk students through the following problem on the board:

Determine the mass of the Sun given that the Earth has an orbital speed of 30 km/s and the distance between the Earth and the Sun is $1.5 \times 10^{\wedge 8}$ km. You may assume that $G=6.67 \times 10^{-20}$ $km^3/(sec^2 * kg)$.

Answer: $2x10^{30}$ kg

6. Students should complete this problem on their own with the teacher circulating:

Using your answer to the previous problem, solve for the orbital speed of Jupiter given that its distance from the Sun is 7.8×10^8 km.

Answer: 13 km/s

7. Students should work individually but can discuss their answers with a partner:

Create a graph of the orbital velocity vs. distance from the Sun from the three data points for Earth, Mars, and Jupiter (for Mars, the orbital speed is 24 km/s and the distance to the Sun is 2.3 x 10^8 km). Make the x-axis be distance in units of 1×10^8 km up to 20 $\times 10^8$ km. Make the y-axis be velocity in units of km/s up to 35. Solve the Motions Find Mass equation algebraically for V as a function of the other variables. Discuss how the shape of the graph compares to the form of the equation. Use the graph to predict the orbital velocity of Saturn given that it orbits 13.5x10^8 km from the Sun. Compare with the real value *[teacher should provide after students have had a chance to determine answer: 9.7 km/s]*. Modify the curve on the graph to include the new data point.

Enrichment

If a person is standing on the surface of the Earth, he/she is rotating along with the surface. Can you use the person's velocity and distance from the center of the Earth to determine the Earth's mass? Discuss. [Students must distinguish rotation of rigid body from orbital rotation.]

Post-Lesson

8. In each situation, how can you maintain a stable circular orbit?

(c) the velocity of the orbiting object increases, how must its orbital radius change if the central mass is the same?

(d) the radius of the orbiting object decreases, how must the mass of the central object change if the orbital velocity is the same?

9. An asteroid has an orbital velocity of 18.2 km/s and is located near the middle of the asteroid belt. Draw on your graph from class to predict the radius of the asteroid's orbit around the Sun. Now solve the Motions Find Mass equation for R and compute the radius of the orbit. Compare your prediction and your calculation. [*teacher's note: answer is* 4×10^8 *km*]. Now consider another asteroid with twice the mass of the first one. Compare their orbital velocities assuming both are located near the middle of the asteroid belt. [*teacher's note: orbital velocities are the same, because they do not depend on the orbiting body's mass*]

10. Which of the following influence the orbital speed of the Moon?

a) the radius of the Earth
b) the radius of the Moon
c) the mass of the Earth
d) the mass of the Moon

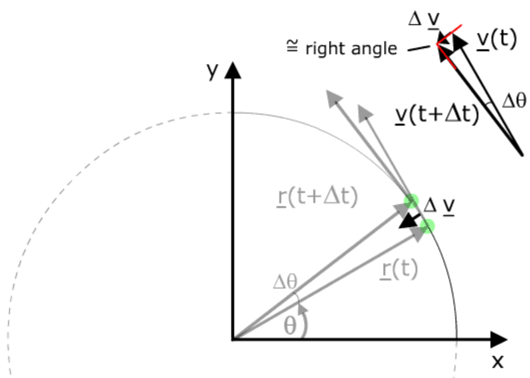
Explain your answer.

Appendix: Derivation of Centripetal Force Equation

You may have learned that the equation for centripetal force is $F=mv^2/R$. Is this a law? What would make it a law?

We usually think of laws as empirical truths that always hold, such as Newton's Second Law $F=ma$. In contrast $F=mv^2/R$ is not true of just any object moving with velocity v at radius R from another point. Rather, this equation is derived under certain assumptions: the object should be in constant-speed circular motion.

Once we assume those two things, they *imply* the centripetal force equation via the following derivation.



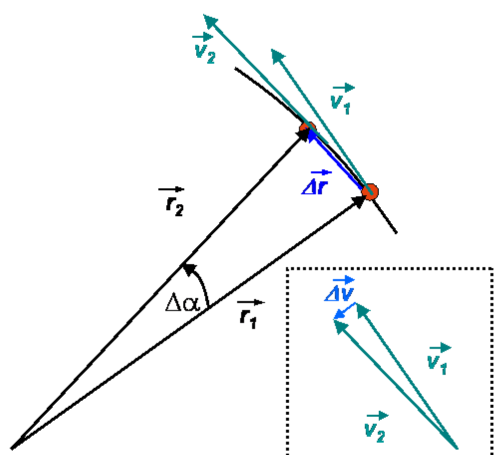
First, note that from our assumptions, the object's speed v isn't changing, but the direction of motion is changing. That means by definition the object is accelerating, because its velocity <u>vector</u> is changing by Δv (see diagram at left).

By definition:
$v=\Delta r/\Delta t$ and $a=\Delta v/\Delta t$

Now if we want to measure the change in a very small amount of time Δt, we should realize that the Δv is almost perfectly toward the center of the circle, as illustrated in the diagram. This means from the geometry of similar triangles:

$\Delta v/v=\Delta r/r$

If we combine this equation with the two definitions above, we get:
$a = \Delta v/\Delta t = (v\Delta r/r)/\Delta t = v^2/r$



Look back: why does this derivation break down without the assumption of constant circular motion?